In the canonical ensemble, a system at equilibrium with a heat bath at temperature T is described using the partition function, denoted by Z(), where, is the inverse temperature[[1]](#footnote-1).

For a quantum system with discrete eigenstates and corresponding eigenvalues , the canonical partition function is given by,

Where, is the system Hamiltonian and the trace is taken over the complete Hilbert space of the system.

The Helmholtz free energy is related to the partition function via,

The derivation will begin with the definition of the average internal energy of the system:

The entropy for the system is just the von Neumann entropy and is given by,

Where the density operator is given by,

Now, we have all of the required quantities for the derivation. Therefore, we will plug the density operator equation in the von Neumann entropy expression to obtain,

Now, from the definition of the total energy of the system,

We will isolate the natural log and obtain,

1. It serves as a convenient parameter for expressing thermal expectation values and derivatives. [↑](#footnote-ref-1)